Stockholm University
Department of Statistics
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Econometrics II

WRITTEN EXAMINATION
Wednesday August 12, 2015, 15 am - 20 pm

Tools allowed: Pocket calculator

Passing rate: 50% of overall total, which is 100 points. For detailed grading criteria, see the course description.

For the maximum number of points on each problem detailed and clear solutions are required.

Observe:: If not indicated otherwise, the error terms ϵ_i in the models are assumed independent and $N(0, \sigma^2)$.

Det finns ingen svensk version den här gången, men om det finns språkliga oklarheter, så är det bara att fråga under tentamen. Lösningarna får ges på svenska.

1. (30p) Some smoothing related problems:

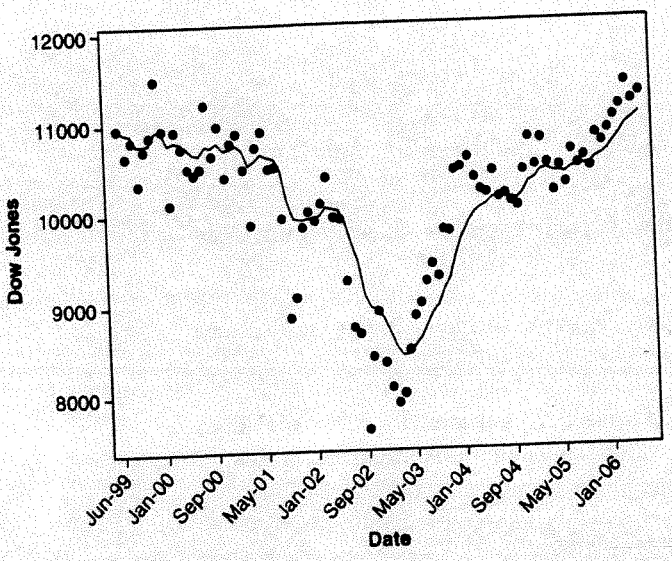
   (a) Below we have a second-order exponential smoothing of U.S. consumer price index, where some values are missing. Fill in the missing values.

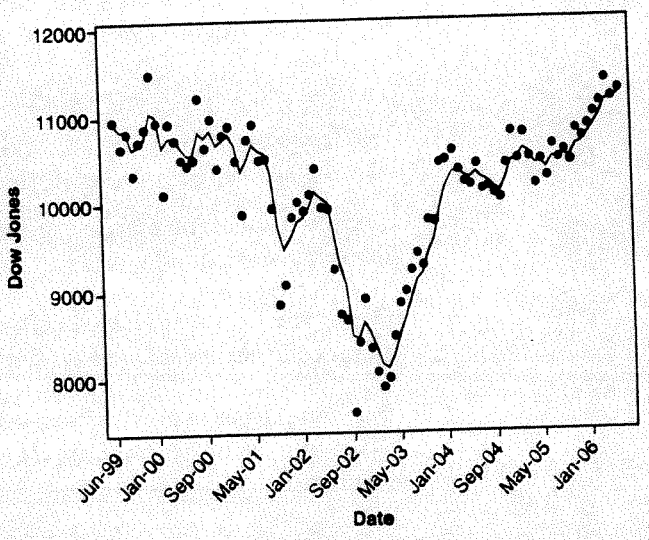
Second-Order Exponential Smoothing of the U.S. Consumer Price Index (with $\lambda = 0.3$, $\tilde{y}_0^{(1)} = y_1$, and $\tilde{y}_0^{(2)} = \tilde{y}_1^{(1)}$)

Date	y_t	$\tilde{y}_T^{(1)}$	$\tilde{y}_T^{(2)}$	$\hat{y}_T$
Jan-1995	150.3	150.300	150.300	150.300
Feb-1995	150.9			
Mar-1995	151.4			
Apr-1995	151.9	151.099	150.662	151.536
May-1995	152.2	151.429	150.892	151.967
⋮				
Nov-2004	191.0	190.041	188.976	191.106
Dec-2004	190.3	190.119	189.319	190.919

(10p)

(b) Consider the following two figures of the Dow Jones Index, where first-order exponential smoothers are included, using different values of the discount factor λ. In one of the figures $\lambda = 0.2$ and in the other $\lambda = 0.4$, but which is which? Decide this with motivation.

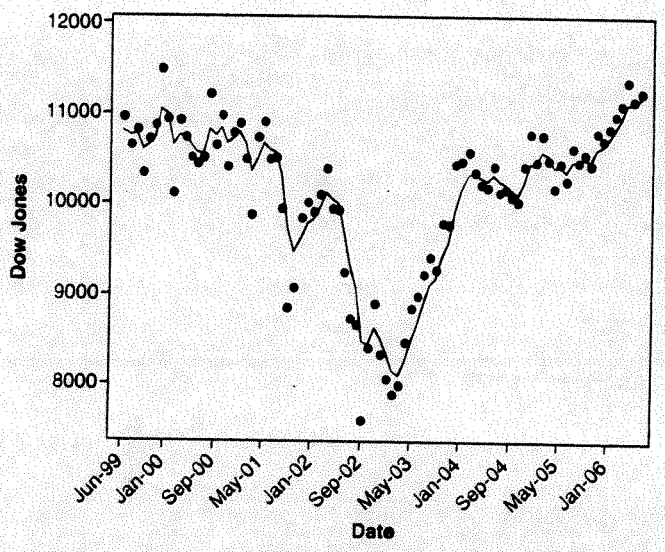


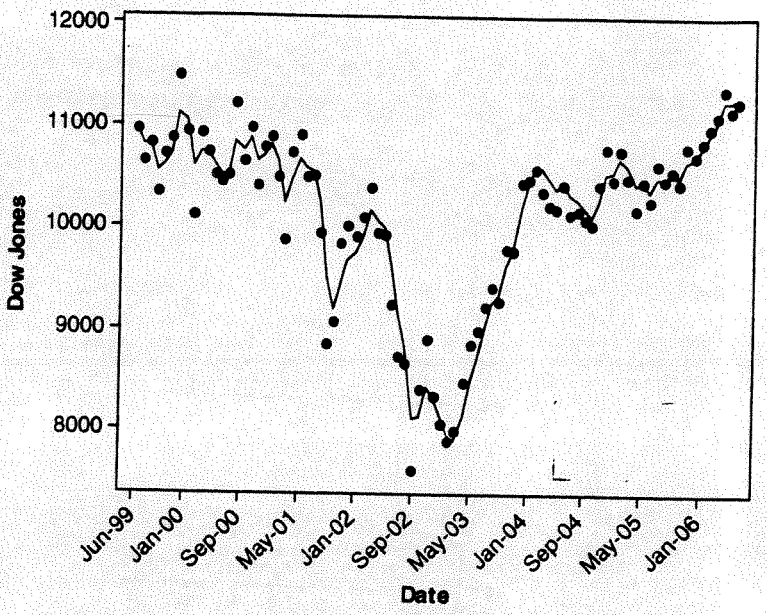


(10p)

(c) Now we consider two figures where in one figure we have incuded a first-order exponential smoother and in the other a second-order exponential smoother. Also here the question is: which is which? Decide with motivation.

Also mention **one** advantage of using second-order smoothing instead of first-order smoothing





(10p)

2. We have different types of data structures within econometrics: cross-sectional, time series and panel data. Give examples of separate situations where we use these three types of data. (You do not have to provide numerical values, but examples of variables we can use.) Also mention **one** example of potential problem when trying to fit a model using panel data.

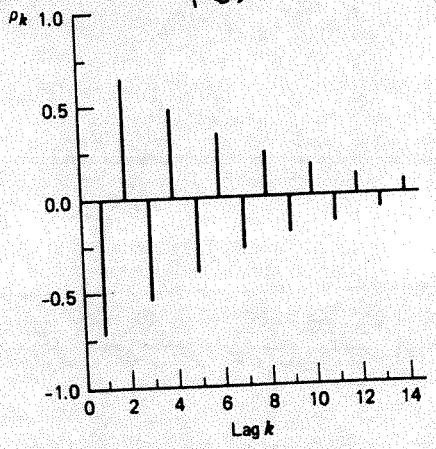
(10p)

3. Below we have autocorrelation values for the following different AR(2) processes:

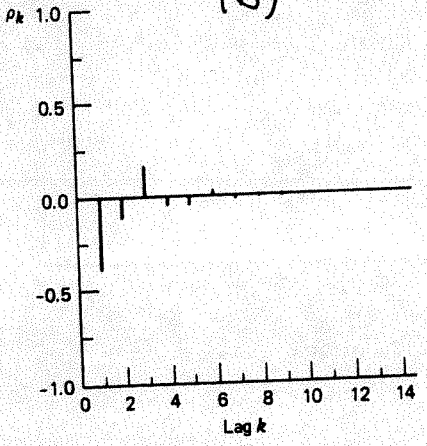
$$(1): \ \phi_1 = 0.5, \ \phi_2 = 0.3$$
$$(2): \ \phi_1 = 1.0, \ \phi_2 = -0.5$$
$$(3): \ \phi_1 = -0.5, \ \phi_2 = 0.3$$
$$(4): \ \phi_1 = -0.5, \ \phi_2 = -0.3$$

Combine (with motivation) each process with one of the figures below. (Each figure corresponds to one of the above processes.)

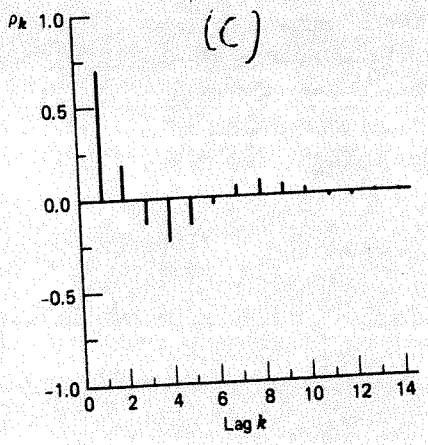
(a)



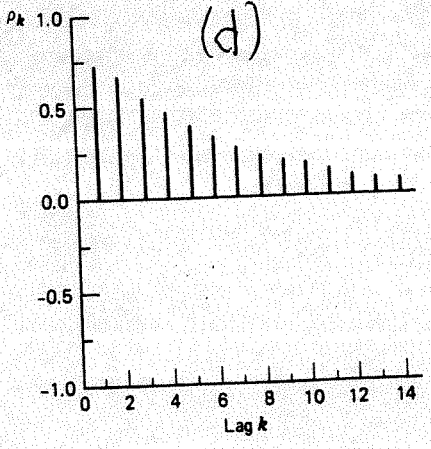
(b)



(c)



(d)



(10p)

4. Consider the following AR(2) process:

$$y_t = 3 + 0.7y_{t-1} + 0.2y_{t-2} + \epsilon_t$$

(a) Without solving it, show which equation you should put up in order to investigate if the process is stationary.
(b) Verify that it **is** stationary.
(c) Compute $E(y_t)$, ρ_1, ρ_2 and ρ_3.

(15p)

Consider the following process:

$$y_t = \delta + y_{t-1} + 0.3y_{t-2} - 0.6\epsilon_{t-1} - 0.8\epsilon_{t-2} + \epsilon_t$$

(a) Which process is this?
(b) Show that it is not stationary.
(c) Determine which ARIMA-model this is and write $w_t = \ldots$.
(d) Show that w_t is stationary.

(15p)

5. Some random walk problems:

(a) Let us first consider the most simple random walk model:

$$y_t = y_{t-1} + \epsilon_t \quad (1)$$

This means that, for example, $y_1 = y_0 + \epsilon_1$. Rewrite (1) so that it is a sum of the starting value y_0 and a stochastic trend. Use this to show that y_t is not stationary.
If we realize values of this process over time, what is the effect of the violation against stationarity? (You can illustrate it graphically if you prefer that.)

(10p)

(b) Now we add a constant δ to our model:

$$y_t = \delta + y_{t-1} + \epsilon_t \quad (2)$$

What is δ ususally called? Rewrite (2) in the same way as you did for (1), so that the result now is that y_t is the sum of the starting value y_0, some function of δ and a stochastic trend.
Show that y_t is nonstationary.
What is the function of δ usually called? What effect has this function on the process? Also here you can illustrate it graphically.)

(10p)

Formula sheet, Econometrics II

Under the simple linear model $y_t = \beta_1 + \beta_2 x_t + u_t$, where $u_t \sim N(0, \sigma^2)$ and given independent pairs of observations $(y_1, x_1), \ldots, (y_n, x_n)$, the OLS (and ML) estimators are:

$$\begin{aligned}
\widehat{\beta}_1 &= \overline{y} - \widehat{\beta}_2 \overline{x} \\
\widehat{\beta}_2 &= \frac{\sum (x_t - \overline{x})(y_t - \overline{y})}{\sum (x_t - \overline{x})^2} \\
\widehat{\sigma}^2 &= \frac{RSS}{n-2} = \frac{\sum (y_t - \widehat{y}_t)^2}{n-2}
\end{aligned}$$

where $\widehat{y}_t = \widehat{\beta}_1 + \widehat{\beta}_2 x_t$ and where $E(\widehat{\beta_1}) = \beta_1$, $E(\widehat{\beta_2}) = \beta_2$ and $E(\widehat{\sigma}^2) = \sigma^2$

Comparing an "old" model with a "new" (larger):

$$\begin{aligned}
F &= \frac{(ESS_{new} - ESS_{old})/\text{number of new regressors}}{RSS_{new}/(n - \text{number of parameters in the new model})} \\
&= \frac{(R^2_{new} - R^2_{old})/\text{number of new regressors}}{(1 - R^2_{new})/(n - \text{number of parameters in the new model})}
\end{aligned}$$

Comparing an "unrestricted" model with a "restricted":

$$F = \frac{(RSS_R - RSS_{UR})/m}{RSS_{UR}/(n-k)} = \frac{(R^2_{UR} - R^2_R)/m}{(1 - R^2_{UR})/(n-k)}$$

where m is the number of linear constraints and k is the number of parameters in the unrestricted model.

Dynamic models: $y_t = \alpha_0 + \alpha_1 x_t + \alpha_2 y_{t-1} + v_t$

Koyck: $y_t = \alpha(1-\lambda) + \beta_0 x_t + \lambda y_{t-1} + v_t$
Adaptive expectations: $y_t = \gamma\beta_0 + \gamma\beta_1 x_t + (1-\gamma) y_{t-1} + (u_t - (1-\gamma) u_{t-1})$
Partial adjustment: $y_t = \delta\beta_0 + \delta\beta_1 x_t + (1-\delta) y_{t-1} + \delta u_t$

The Durbin Watson d statistic:

$$d = \frac{\sum_{t=2}^{n} (\widehat{u}_t - \widehat{u}_{t-1})^2}{\sum_{t=1}^{n} \widehat{u}_t^2}$$

The Durbin h statistic:

$$h = \widehat{\rho} \sqrt{\frac{n}{1 - n\,[var(\widehat{\alpha}_2)]}} \approx N(0,1), \text{ if } \rho = 0$$

$$MSE = \frac{1}{n}\sum_{t=1}^{n}\left[e_t(t-1)\right]^2 = \frac{1}{n}\sum_{t=1}^{n}\left[y_t - \widehat{y}_t(t-1)\right]^2$$

Autocorrelation function:

$$\rho_k = \frac{Cov(y_t, y_{t+k})}{V(y_t)}, \quad k = 0, 1, 2, \ldots$$

Sample correlation function:

$$\hat{\rho}_k = \frac{\sum_{t=1}^{n-k}\left(y_t - \bar{y}\right)\left(y_{t+k} - \bar{y}\right)}{\sum_{t=1}^{n-k}\left(y_t - \bar{y}\right)^2}, \quad k = 0, 1, 2, \ldots$$

Simple moving average:

$$M_T = \frac{1}{N}\sum_{t=T-N+1}^{T} y_t$$

First-order exponential smoothing:

$$\widetilde{y}_T = \lambda y_T + (1-\lambda)\,\widetilde{y}_{T-1}$$

Second-order exponential smoothing:

$$\widetilde{y}_T^{(2)} = \lambda \widetilde{y}_T^{(1)} + (1-\lambda)\widetilde{y}_{T-1}^{(2)},$$

where $\tilde{y}_0^{(2)} = \tilde{y}_1^{(1)}$

Holt's method:

$$\begin{aligned} L_t &= \alpha y_t + (1-\alpha)\left(L_{t-1} + T_{t-1}\right) \\ T_t &= \gamma\left(L_t - L_{t-1}\right) + (1-\gamma)\,T_{t-1} \end{aligned}$$

$$\widehat{y}_{T+\tau}(T) = L_T + \tau T_T, \quad \tau = 1, 2, \ldots$$

Forecast under a constant process:

$$\widehat{y}_{T+\tau}(T) = \widetilde{y}_T \quad \tau = 1, 2, ...$$

Forecast under a linear trend:

$$\hat{y}_{T+\tau}(T) = \hat{y}_T + \hat{\beta}_{1,T}\tau,$$

where $\hat{y}_T = \hat{\beta}_{0,T} + \hat{\beta}_{1,T}T = 2\tilde{y}_T^{(1)} - \tilde{y}_T^{(2)}$

The Ljung-Box statistic;:

$$Q_{LB} = T(T+2)\sum_{k=1}^{K}\left(\frac{\hat{\rho}_k^2}{T-k}\right) \approx \chi^2(K)$$

ARMA(p,q):

$$y_t = \delta + \sum_{i=1}^{p}\phi_i y_{t-i} + \epsilon_t - \sum_{i=1}^{q}\theta_i\epsilon_{t-i}$$

Stationarity and invertibility conditions for some time series models:

Model	Stationarity conditions	Invertibility conditions
AR(1)	$\lvert\phi_1\rvert < 1$	None
AR(2)	$\phi_1 + \phi_2 < 1$ $\phi_2 - \phi_1 < 1$ $\lvert\phi_2\rvert < 1$	None
MA(1)	None	$\lvert\theta_1\rvert < 1$
MA(2)	None	$\theta_1 + \theta_2 < 1$ $\theta_2 - \theta_1 < 1$ $\lvert\theta_2\rvert < 1$
ARMA(1,1)	$\lvert\phi_1\rvert < 1$	$\lvert\theta_1\rvert < 1$
ARMA(2,2)	$\phi_1 + \phi_2 < 1$ $\phi_2 - \phi_1 < 1$ $\lvert\phi_2\rvert < 1$	$\theta_1 + \theta_2 < 1$ $\theta_2 - \theta_1 < 1$ $\lvert\theta_2\rvert < 1$

The Yule-Walker equations for AR(p):

$$\rho_k = \sum_{i=1}^{p}\phi_i\rho_{k-i}, \quad k = 1, 2, \ldots$$

Statistiska institutionen

Stockholms universitet

Rättningsblad

Datum: 12/8-2015

Sal: Brunnsvikssalen

Tenta: Tidsserieanalys

Kurs: Ekonometri

ANONYMKOD: EKT-0048

☒ Jag godkänner att min tenta får läggas ut anonymt på hemsidan som studentsvar.

OBS! SKRIV ÄVEN PÅ BAKSIDAN AV SKRIVBLADEN

Markera besvarade uppgifter med kryss

1	2	3	4	5	6	7	8	9	Antal inl. blad
X	X	X	X	X	X				5
Lär.ant. 30	10	10	15	12	20				

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SU, STATISTIK

Skrivsal: Brunnsviksalen Anonymkod: EKT-0048 Blad nr: 7

7.

a/ Consumer Price Index (with $\lambda = 0,3$, $\tilde{y}_0^{(1)} = y_1$, and $\tilde{y}_0^{(2)} = \tilde{y}_1^{(1)}$)

Date	y_t	$\tilde{y}_T^{(1)}$	$\tilde{y}_T^{(2)}$	$\hat{y}_T$
Jan-1995	150,3	150,300	150,3	150,3
Feb-1995	150,9	150,48	150,354	150,606
Mar-1995	151,4	150,756	150,475	151,037
Apr-1995	151,9	151,029	150,662	151,536
May-1995	152,2	151,429	150,892	151,967

t_1 = Jan-1995, t_2 = Feb-1995, etc...

$\tilde{y}_T = \lambda y_T + (1-\lambda)\tilde{y}_{T-1}$

$\Rightarrow \tilde{y}_2 = 0,3 \cdot 150,9 + 0,7 \cdot 150,300 = 150,48$

$\tilde{y}_3 = 0,3 \cdot 151,4 + 0,7 \cdot 150,48 = 150,756$

$\tilde{y}_T^{(2)} = \lambda \tilde{y}_T^{(1)} + (1-\lambda)\tilde{y}_{T-1}^{(2)}$

$\tilde{y}_2^{(2)} = 0,3 \cdot 150,48 + 0,7 \cdot 150,3 = 150,354$

$\tilde{y}_3^{(2)} = 0,3 \cdot 150,756 + 0,7 \cdot 150,354 = 150,475$

$\hat{y}_T = 2\tilde{y}_T^{(1)} - \tilde{y}_T^{(2)}$

$\hat{y}_2 = 2 \cdot 150,48 - 150,354 = 150,606$

$\hat{y}_3 = 2 \cdot 150,756 - 150,475 = 151,037$

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7.

b/ The left one is $\alpha=0,2$ and the right one is $\alpha=0,4$. This is due to the fact that the closer α is to 1, the closer it follows the observed values (if $\alpha=1$ then $\tilde{y}_T=y_T$). The left one is clearly the more smoothed one, and it's therefore $\alpha=0,2$. /10

c/ There seems to be a trend in the data and the estimations on the left picture seem to be biased. When there is a trend in the data, first-order exponential smoothers tend to be biased.
Therefore the left picture is first-order and the right one is second-order exponential smoother. An advantage of second-order over first-order exponential smoothing is that it can be used under linear trend. /10

SU, STATISTIK

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2.

i) We can use cross-sectional data to perform regression analysis when we have data from for example different actors at the same time point.
Ex. Regressing Y_i = Country's GDP on X_{ii} = Country's natural resources

ii) Time-series data can be useful when we want to explore one object's behaviour across different time points.
Ex. a stock option Y_t = Price at time point t, ε_t = error term
$Y_t = Y_{t-1} + \varepsilon_t$

iii) Panel data is when we combine cross-sectional and time series data. We can for example research different airlines at different time points. and therefore get time points x airlines amount of observations.
Ex. Y_{it} = Total costs of airline i at time t, X_{1it} = Fuel cost of airline i at t,
One problem with modelling panel data is that the error terms may be autocorrelated, which has to be taken into account.

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3. We use the Yule-Walker equations to calculate ρ_1 and ρ_2 for each process and then compare them to the graphs:

$$\rho_k = \sum_{i=1}^{p} \phi_i \rho_{k-i}, \quad k = 1, 2, \ldots$$

$$\rho_1 = \phi_1 \underbrace{\rho_0}_{1} + \phi_2 \underbrace{\rho_{-1}}_{\rho_1} \Rightarrow (1-\phi_2)\rho_1 = \phi_1 \Rightarrow \rho_1 = \frac{\phi_1}{1-\phi_2}$$

$$\rho_2 = \phi_1 \rho_1 + \phi_2 \underbrace{\rho_0}_{1} \Rightarrow \rho_2 = \phi_1 \rho_1 + \phi_2$$

(1) $\rho_1 = \frac{0,5}{1-0,3} \approx 0,714 \quad \rho_2 = 0,5 \cdot 0,714 + 0,3 \approx 0,657$

$\Rightarrow$ (1) $\approx$ (d)

(2) $\rho_1 = \frac{1}{1-(-0,5)} = \frac{1}{1,5} \approx 0,667 \quad \rho_2 = 1 \cdot 0,667 + (-0,5) \approx 0,167$

$\Rightarrow$ (2) $\approx$ (c)

(3) $\rho_1 = \frac{-0,5}{1-0,3} \approx -0,714 \quad \rho_2 = -0,5 \cdot (-0,714) + 0,3 \approx 0,657$

$\Rightarrow$ (3) $\approx$ (a)

(4) $\rho_1 = \frac{-0,5}{1+0,3} \approx -0,385 \quad \rho_2 = -0,5(-0,385) + 0,3 \approx -0,108$

$\Rightarrow$ (4) $\approx$ (b)

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SU, STATISTIK

Skrivsal: Brunnsviksalen Anonymkod: EKT-0048 Blad nr: 4

4.

a) If both of the roots m_1 & m_2 to the equation $m^2 - \phi_1 m - \phi_2 = 0$ $\Rightarrow m^2 - 0,7m - 0,2 = 0$ are under 1 in absolute value, we have stationarity.

b) Stationarity conditions for AR(2) process:

$\phi_1 + \phi_2 < 1$, $\phi_2 - \phi_1 < 1$, $|\phi_2| < 1$

$\phi_1 + \phi_2 = 0,7 + 0,2 = 0,9$

$\phi_2 - \phi_1 = 0,2 - 0,7 = -0,5$

$|\phi_2| = 0,2$

All under 1 $\Rightarrow$ Stationary

c) $E(y_t) = E(3 + 0,7y_{t-1} + 0,2y_{t-2} + \epsilon_t)$

Since we have stationarity, $E(y_t) = E(y_{t-1}) = E(y_{t-2}) = \mu$ and $\epsilon_t \sim N(0, \sigma^2)$:

$\Rightarrow \mu = 3 + 0,7\mu + 0,2\mu + 0$

$\Rightarrow \mu(1 - 0,7 - 0,2) = 3 \Rightarrow \mu = \frac{3}{0,1} = \underline{\underline{30}}$

Yule-Walker equations for AR(p): $\rho_k = \sum_{i=1}^{p} \phi_i \rho_{k-i}$, $k = 1, 2, \ldots$

$\rho_1 = \frac{\phi_1}{1 - \phi_2}$, $\phi_1 = 0,7$, $\phi_2 = 0,2$

$\rho_1 = \frac{0,7}{0,8} \approx 0,875$

$\rho_2 = \phi_1 \rho_1 + \phi_2 \approx 0,7 \cdot 0,875 + 0,2 \approx 0,813$

$\rho_3 = \phi_1 \rho_2 + \phi_2 \rho_1 = 0,7 \cdot 0,813 + 0,2 \cdot 0,875 \approx 0,744$ //5

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5.

a) It is ARMA(2,2)

b) Stationarity conditions for ARMA(2,2):

$\phi_1 + \phi_2 < 1$, $\phi_2 - \phi_1 < 1$, $|\phi_2| < 1$

$\phi_1 = 1$, $\phi_2 = 0,3$

$\phi_1 + \phi_2 = 1,3 \Rightarrow > 1$, therefore not stationary

c) We can transform this process to ARIMA by transforming

$w_t = y_t - y_{t-1}$

$\Rightarrow w_t = \delta + 0,3 y_{t-2} - 0,6 \varepsilon_{t-1} - 0,8 \varepsilon_{t-2} + \varepsilon_t$

Which is ARIMA(2, 1, 2)

d) Since now

$\phi_1 + \phi_2 = 0,3$

$\phi_2 - \phi_1 = 0,3$

$|\phi_2| = 0,3$

Sufficient here to check the AR-coefficients (=0.3)

All under 1 $\Rightarrow w_t$ is stationary

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6.

a) $y_t = y_{t-1} + \varepsilon_t$

$\Rightarrow y_t = y_{t-2} + \varepsilon_t + \varepsilon_{t-1}$

$\Rightarrow y_t = y_{t-3} + \varepsilon_t + \varepsilon_{t-1} + \varepsilon_{t-2}$

$\Rightarrow y_t = y_0 + \sum_{k=0}^{t-1} \varepsilon_{t-k}$

Since $V(y_t) = V(y_0 + \sum_{k=0}^{t-1} \varepsilon_{t-k}) = \sum_{k=0}^{t-1} V(\varepsilon_{t-k}) = t\sigma^2$, $(\varepsilon_t \sim N(0, \sigma^2))$

is dependent on time, we have non-stationarity.

- Variance grows over time

b) δ is the drift component in random walk with drift.

$y_t = \delta + y_{t-1} + \varepsilon_t$

$\Rightarrow y_t = 2\delta + y_{t-2} + \varepsilon_t + \varepsilon_{t-1}$

$\Rightarrow y_t = y_0 + t\delta + \sum_{k=0}^{t-1} \varepsilon_{t-k}$

$V(y_t) = V(y_0 + t\delta + \sum_{k=0}^{t-1} \varepsilon_{t-k})$

$= \sum_{k=0}^{t-1} V(\varepsilon_{t-k}) = t\sigma^2$, same as in a)

$\Rightarrow$ nonstationarity

The function of δ is the deterministic trend. This causes the mean of the process to shift each time period.

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